B-Tech C.S.E Sem-I (2021-course) winter-2022

BACHELOR OF TECHNOLOGY (C.B.C.S.) (2021-COURSE)
B. Tech. Sem - I Computer Science & Engineering : WINTER- 2022
SUBJECT : MATHEMATICS FOR COMPUTING-I

Day : Monday | Time : 10:00 AM-01:00 PM

Date : 9/1/2023 | **W-24018-2022** | Max. Marks : 60

N.B:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.
4) Assume suitable data if necessary.

Q.1 Reduce the matrix $A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & -7\end{bmatrix}$ to an echelon form and further reduce A to its row canonical form. **[10]**

OR

Q.1 Find LU factorization of **[10]**

$$A=\begin{bmatrix}1 & -3 & 5\\ 2 & -4 & 7\\ -1 & -2 & 1\end{bmatrix}$$

Q.2 Let W be the subspace of R^5 spanned by **[10]**
$u_1 = (1, 2, -1, 3, 4)$, $u_2 = (2, 4, -2, 6, 8)$, $u_3 = (1, 3, 2, 2, 6)$, $u_4 = (1, 4, 5, 1, 8)$, $u_5 = (2, 7, 3, 3, 9)$.
Find a subset of the vector that form a basis of W.

OR

Q.2 Examine for linear dependent or independent the following system of vectors. If dependent, find relation between them. **[10]**
$u_1= (2, 2, 7, -1)$, $u_2 = (3, -1, 2, 4)$, $u_3 = (1, 1, 3, 1)$.

Q.3 Show that linear operator T on R^3 is nonsingular and find a formula for T^{-1} where $T(x, y, z) = (x + z, x - y, y)$. **[10]**

OR

Q.3 Let $F: R^5 \rightarrow R^3$ be linear map defined by **[10]**
$F(x, y, z, s, t) = (x+2y+2z+s+t,\ x+2y+3z+2s-t, 3x+6y+8z+5s-t)$.
Find basis and dimension of image of F and kernel of F.

Q.4 The vectors $u_1 = (1, 2, 0)$, $u_2 = (1, 3, 2)$, $u_3 = (0, 1, 3)$ form basis S of R^3. Find the change of basis matrix P from the usual basis $E = \{e_1, e_2, e_3\} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ to S. Find the change of basis matrix Q from S back to E. **[10]**

P.T.O.

OR

Q.4 Consider the linear transformation F on R^2 defined by $F(x, y) = (5x - y,\ 2x + y)$ and the following basis of R^2. **[10]**
$E = \{e_1, e_2\} = \{(1, 0), (0, 1)\}$; $S = \{u_1, u_2\} = \{(1, 4), (2, 7)\}$.
Find the matrix A that represents F in the basis E. Find the matrix B that represents F in the basis S.

Q.5 Consider the subspace U of R^4 spanned by the vectors: **[10]**
$v_1 = (1, 1, 1, 1)$, $v_2 - (1, 1, 2, 4)$, $v_3 = (1, 2, -4, 3)$
Use Gram-Schmidt orthogonolization, to find orthogonal basis of U and an orthonormal basis of U.

OR

Q.5 Consider the vector space $P_2(t)$ with inner product $\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)dt$. Find $\langle f, g\rangle$, where $f(t) = t + 2$, $g(t) = t^2 - 3t + 4$. Find the matrix A of the inner product with respect to the basis $\{1, t, t^2\}$. **[10]**

Q.6 Find the Eigen values and Eigen vectors of matrix A, when **[10]**

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

OR

Q.6 Verify Cayley-Hamilton theorem for the matrix A and use it to find A^4 and A^{-1}. **[10]**

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 2 & 5 & 2 & -1 \\ 0 & 1 & 18 & -7 \\ 0 & -26 & 12 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$