ADDITIONAL EXAM. COMMON FOR ALL BRANCHES
B.TECH. SEM. - I (CBCS 2014 COURSE) : WINTER- 2019
SUBJECT: ENGINEERING MATHEMATICS - I

Monday 23/12/2019 W-11247-2019
10:00 AM-01:00 PM Max. Marks: 60

N.B.:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.
4) Assume suitable data if necessary.

Q.1 Find the Eigen values and the corresponding Eigen vectors of the following matrix. **[10]**

$$A=\begin{bmatrix}-2 & 2 & 3\\ 2 & 1 & 6\\ 3 & 6 & 6\end{bmatrix}$$

OR

a) Solve the following system of equations: **[05]**

$$x+2y+3z=0$$
$$2x+3y+z=0$$
$$4x+5y+4z=0$$
$$x+y-2z=0$$

b) Examine for linear dependence or independence the following vectors: **[05]**

$V_1=(1,-1,1)$ $\quad V_2=(2,1,1)$ $\quad V_3=(3,0,2)$

Q.2 The center of a regular hexagon is $(2-i)$ and one of its vertices is $(-1+i)$. Find the two adjacent vertices of the hexagon. **[10]**

OR

a) If $\frac{z-1}{z+i}$ is purely imaginary, find the locus of z. **[05]**

b) Find the cube roots of unity. **[05]**

Q.3 If $I_n=\frac{d^n}{dx^n}\left(x^n\log x\right)$, prove that $I_n=nI_{n-1}+(n-1)!$. Also show that **[10]**

$$I_n=(n!)\left[\log x+1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right].$$

OR

a) Prove that $\log(\sec x)=\frac{x^2}{2}+\frac{1}{3}\frac{x^4}{4}+\frac{2}{15}\frac{x^6}{6}+....$ **[05]**

b) Expand : $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in ascending powers of x. **[05]**

P.T.O.

Q.4 a) Evaluate: $\lim_{x \to 1} \frac{\sqrt{x} - 1 + \sqrt{x-1}}{\sqrt{x^2 - 1}}$. **[05]**

b) Test the convergence of the following series: **[05]**

$$\frac{1^1}{2^2} + \frac{1^2.3^2}{2^2.4^2} + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \dots\dots$$

OR

a) Evaluate: $\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$. **[05]**

b) Test the convergence of the series: **[05]**

$$1 + \frac{2^2}{3^2} + \frac{2^2.4^2}{3^2.5^2} + \frac{2^2.4^2.6^2}{3^2.5^2.7^2} + \dots\dots$$

Q.5 If f(x, y) and ϕ (x, y) are homogeneous functions of x, y of degree p, q respectively and $u = f(x,y) + \phi(x,y)$. Show that **[10]**

$$f(x,y) = \frac{1}{p(p-q)} \left[x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \right] - \frac{q-1}{p(p-q)} \left[x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right]$$

OR

If $u = \frac{x^2 y^2 z^2}{x^2 + y^2 + z^2} + \cos\left(\frac{xy + yz}{x^2 + y^2 + z^2} \right)$, show that **[10]**

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 4 \left(\frac{x^2 y^2 z^2}{x^2 + y^2 + z^2} \right).$$

Q.6 a) The density ρ of a body is calculated from its weight W in air and w in water. If errors dW and dw are made in W and w. Find the relative error in ρ. **[05]**

b) Examine for functional dependence $u = \frac{x-y}{x+y}$, $v = \frac{x+y}{x}$, if functionally dependent find the relation between them. **[05]**

OR

Discuss the maxima and minima of the following function. **[10]**

$$f(x,y) = x^2 y^2 - 5x^2 - 8xy - 5y^2.$$

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