

BACHELOR OF TECHNOLOGY (CBCS) (2021-COURSE)
B. Tech. Sem - I Computer Science & Business Systems : WINTER : 2023
SUBJECT : DISCRETE MATHEMATICS

Day : Thursday
Date : 30-11-2023

W-24130-2023

Time : 10:00 AM-01:00 PM
Max. Marks : 60

N.B.

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.

- Q.1** Define logical equivalence and contingency of a statement. State converse, (10)
inverse and contrapositive of the following:
- a) If today is Easter, then tomorrow is Monday
 - b) If a triangle is not isosceles, then it is not equilateral.
 - c) If n is prime then n is odd or n is 2.

OR

Define Duality Law and find conjunctive normal forms of :

- i) $((\neg p \rightarrow r) \wedge (p \rightarrow q))$ ii) $(p \wedge q) \vee (\neg p \wedge q \wedge r)$

- Q.2** Prove the expressions by using the Venn diagrams: (10)
- a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 - b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 - c) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$

OR

Out of integers 1 to 1000 :

- a) How many are not divisible by 3 nor by 5 nor by 7?
- b) How many are not divisible by 5 and 7 but divisible by 3?

- Q.3** Let $A = B$ be the set of real numbers: (10)
- $f : A \rightarrow B$ given by $f(x) = 2x^3 - 1$
- $g : B \rightarrow A$ given by $g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$
- Show that f is bijection between A and B and g is bijection between B and A .

OR

Define poset with suitable example. Also find transitive closure of R by
Warshall's algorithm where
 $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(x, y) \mid (x - y) = 2\}$

- Q.4** Explain properties of binary operations and prove the following results for a (10)
Group G :
- a) The identity element is unique
 - b) $ab = ac$ implies $b = c$.

P.T.O.

OR

Define:

- a) Subgroup
- b) Abelian group
- c) Integral Domains
- d) Rings

Show that, $S = (a + b\sqrt{2}; a, b \in \mathbb{Z})$ for operation $+$, $*$ is an integral domain.

- Q.5 Solve the recurrence relations $b_n = -3b_{n-1} - 2b_{n-2}$ with initial condition (10)
 $b_1 = -2, b_2 = 4.$

OR

Three electric bulbs are chosen at random from 15 bulbs of which 5 are defective:

Find the probability that:

- a) None is defective
- b) Exactly one is defective
- c) Atleast one is defective

- Q.6 Define:

- a) Complete Graph
- b) Regular graph
- c) Bipertile graph
- d) Isomorphic graph

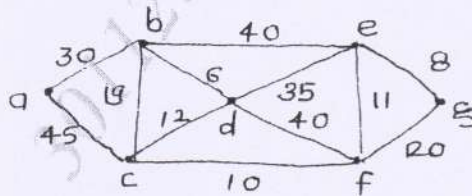
(10)

Show that $K_{n,n}$ is Hamiltonian if and only if $n \geq 2$ hence show that, $K_{n,n}$ has

$$\frac{n!(n-1)!}{2} \text{ cycles.}$$

OR

Find the shortest path between a and g in the given weighted graph.



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