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B.Tech. sem-I all Branch

B.Tech. SEM -I (Chemical/ Civil/ Electrical/ Mechanical/ Production/ Computer/ Info. Tech./ Electronics / Bio Medical / E & TC) 2014
Course (CBCS) : SUMMER - 2019
SUBJECT: ENGINEERING MATHEMATICS - I

Day: Thursday
Date: 09/05/2019

S-2019-2525

Time: 10.00 AM TO 01.00 PM
Max Marks. 60

N.B. :

1) All questions are **COMPURSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.
4) Draw neat and labeled diagram **WHEREVER** necessary.

Q.1 **a)** Find the rank of the following matrix **(05)**

$$\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$$

b) Examine for linear dependence or independence for the following system of vectors. **(05)**

$x_1 = (3, 1, -4)$, $x_2 = (2, 2, -3)$, $x_3 = (0, -4, 1)$

OR

Q.1 Find the eigen values and the corresponding eigen vectors of the following matrix. **(10)**

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Q.2 **a)** A square lies entirely in second quadrant if one of the side join the points -2 and 2i find the complex numbers representing other vertices. **(05)**

b) If Z_1, Z_2, Z_3 represent vertices of an equilateral triangle, prove that **(05)**

$$Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$$

OR

Q.2 **a)** Find the value of $\tanh\left(\log(\sqrt{5})\right)$ **(05)**

b) If $\tan(A + iB) = x + iy$ then prove that $x^2 + y^2 + 2x \cot 2A = 1$ **(05)**

Q.3 **a)** If $y = \cos(m \log x)$, show that $x^2 y_{n+2} + (2n+1) x y_{n+1} + (m^2 + n^2) y_n = 0$ **(05)**

b) If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that $(x^2 - 1) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$

OR

Q.3 Prove that the value of the n^{th} differential coefficient of $\frac{x^3}{(x^2 - 1)}$ for $x = 0$ is zero if n is even and $-n!$ if n is odd and greater than 1 **(10)**

P.T.O.

Q.4	**a)**	Evaluate $\lim_{x \to 0} \dfrac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}$	**(05)**
	b)	Test the convergence of the series: $\dfrac{1}{1.3}+\dfrac{2}{3.5}+\dfrac{3}{5.7}+\dfrac{4}{7.9}+\ldots$	**(05)**

OR

Q.4	**a)**	Test the convergence of the series: $1+\dfrac{1}{2.3}+\dfrac{1.3}{2.4.5}+\dfrac{1.3.5}{2.4.6.7}+\ldots$	**(05)**
	b)	Find a, b, c if $\lim_{x \to 0} \dfrac{ae^x - be^{-x} - cx}{x - \sin x} = 4$	**(05)**
Q.5	**a)**	If $u = f(\text{r})$, where $\text{r} = \sqrt{x^2+y^2}$ then prove that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2} = f''(\text{r}) + \dfrac{1}{r} f'(\text{r})$	**(05)**
	b)	If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{2x+3y}\right)$ then prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \sin 2u(2\cos 2u - 1)$	**(05)**

OR

Q.5		Find $\dfrac{dy}{dx}+\dfrac{d^2y}{dx^2}$ for $x^4+y^4 = 5a^2xy$.	**(10)**
Q.6	**a)**	If $u^3+v^3+w^3 = x+y+z,\ u^2+v^2+w^2 = x^3+y^3+z^3,\ u+v+w = x^2+y^2+z^2$ then find $\dfrac{\partial(\text{u, v, w})}{\partial(\text{x, y, z})}$	**(05)**
	b)	If the kinetic energy is given by $T = \dfrac{mv^2}{2}$, find approximately the change in T as m changes from 49 to 49.5 and v changes from 1600 to 1590.	**(05)**

OR

Q.6		Examine for minimum and maximum values for $\sin x + \sin y + \sin(x+y)$	**(10)**

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