BACHELOR OF TECHNOLOGY (CBCS) (2021-COURSE)
B. Tech. Sem - II Computer Science and Business Systems : WINTER : 2023
SUBJECT : LINEAR ALGEBRA

Day : Monday
Date : 20-11-2023

W-24136-2023

Time : 10:00 AM-01:00 PM
Max. Marks : 60

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is allowed.

Q.1 Solve the equation: **(10)**

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$$

OR

Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

Q.2 Find the rank of matrix A, where **(10)**

$$A = \begin{bmatrix} 3 & 4 & 1 & 1 \\ 2 & 4 & 3 & 6 \\ -1 & -2 & 6 & 4 \\ 1 & -1 & 2 & -3 \end{bmatrix}$$

OR

Solve the following system of linear equations

$$2x_1 + x_2 + 5x_3 + x_4 = 5$$
$$x_1 + x_2 - 3x_3 - 4x_4 = -1$$
$$3x_1 + 6x_2 - 2x_3 + x_4 = 8$$
$$2x_1 + 2x_2 + 2x_3 - 3x_4 = 2$$

Q.3 Let W be the subspace of R^5 spanned by **(10)**

$U_1 = (1, 2, -1, 3, 4)$, $U_2 = (2, 4, -2, 6, 8)$

$U_3 = (1, 3, 2, 2, 6)$, $U_4 = (1, 4, 5, 1, 8)$

$U_5 = (2, 7, 3, 3, 9)$

Find a subset of the vectors that form a basis of W.

OR

Suppose $v = (1, 3, 5, 7)$. Find the projection of v onto W, where W is the subspace of R^4 spanned by

$U_1 = (1, 1, 1, 1)$, $U_2 = (1, -3, 4, -2)$

P.T.O.

Q.4 Apply Gram-Schmidt process to construct an orthonormal basis for the subspace $W = span(x_1, x_2, x_3)$ of R^4, where **(10)**

$x_1 = (1, 1, -1, 1)$, $x_2 = (1, -1, 1, 5)$, $x_3 = (1, 2, 0, 1)$

OR

Find a QR-factorization of

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ -1 & -1 \\ 0 & 1 \end{bmatrix}$$

Q.5 Find eigen values and eigen vectors of matrix A where **(10)**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

OR

If

$$A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$$

Show that A is Hermitian matrix and iA is a skew-Hermitian matrix.

Q.6 Find a singular value decomposition of **(10)**

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

OR

Give the following data, use principal component analysis to reduce the dimension from 2 to 1

Feature	Example 1	Example 2	Example 3	Example 4
x	4	8	13	7
y	11	4	5	14

* * * *