BACHELOR OF TECHNOLOGY (CBCS) (2021-COURSE)

Computer Science & Business Systems

B. Tech. Sem - I :SUMMER : 2023

SUBJECT : DISCRETE MATHEMATICS

Day : Tuesday | Time : 10:00 AM-01:00 PM

Date : 09-05-2023 | S-24130-2023 | Max. Marks : 60

N.B.

1) All questions are **COMPULSORY**
2) Figure to the right indicate **FULL** marks.

Q.1 Define converse, inverse and contrapositive of a conditional statement. Find converse, inverse and contrapositive of $\forall x\,(x > 3) \rightarrow (x^2 > 9)$ where universal set is R. **(10)**

OR

Prove the following equivalence **(10)**
$\{(p \rightarrow q) \wedge [\sim q \wedge (r \wedge \sim q)]\} \Leftrightarrow \sim(q \vee p)$
Also define tautology, show that for any proposition p, q, r the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)]$ is tautology.

Q.2 Prove by mathematical induction that for all integers $n \geq 1$, **(10)**
$1 + 2 + 3 + 4 + \cdots\cdots\cdots\cdots\cdots\cdots + n = \frac{1}{2}n(n+1)$

OR

Explain De Morgan's law in set theory and prove that for any two sets A *and* B **(10)**
$(i)\overline{(A \cup B)} = \bar{A} \cap \bar{B}$ $(ii)\ \overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Q.3 Define various properties of relation. Let R be the relation in the real numbers defined by $x \leq y$. Determine weather R is an equivalence relation and antisymmetric relation. **(10)**

OR

Explain pigeonhole principle , consider $A = B = C = R$ and Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be defined by $f(x) = x + 9$ and $g(y) = y^2 + 3$ then find $(i)(fof)(a)$ $(ii)(gog)(a)$ $(iii)(fog)(b)$ $(iv)(gof)(b)$ $(v)(gof)(3)$ $(vi)(fog)(-3)$. **(10)**

Q.4 Define monoid with suitable example. Let Z be the set of all integers (i) Show that the operation * on Z defined by $a * b = a + b + 1$ *for all* $a, b \in Z$ satisfies the closure property , associative law and commutative low (i) Find identity element (ii) Define inverse what is the inverse of an integer a ? **(10)**

OR

Consider an algebraic system $(G, *)$ where G is the set of all non -zero real numbers and $*$ defined by $a * b = \frac{ab}{4}$. Show that $(G, *)$ is an abelian group. **(10)**

P.T.O.

Q.5 Define the terms (*i*) Sample Space (*ii*) Event (*iii*) Mutually exclusive Event (*iv*) Independent Event. **(10)**

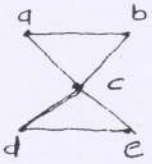
How many 3 digit number can be formed using the digit 2,3,4,5,7 *and* 9 if repetition is not allowed?

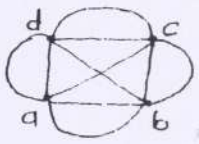
i) How many of these numbers are less than 400
ii) How many are even?
iii) How many are multiples of 5
iv) How many are multiples of 10

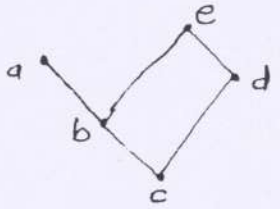
OR

Explain the term probability of an Event and solve the following **(10)**
One card is drawn from a deck of 52 cards if each outcome is equally likely , calculate the probability that the card will be
(i)a spade (ii)a black card (iii)not a spade

Q.6 Define weighted graph with suitable example which of the following graph have a Euler circuit or path or Hamiltonian cycle? Write the path or circuit. **(10)**







OR

Define the following with example with respect to graph theory **(10)**

a) Multigraph.
b) Isomorphic Graph.
c) Bipartite Graph.
d) Self complementary Graph.
e) Planar Graph.

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