

**B.Tech. SEM -I (Chemical/ Civil/ Electrical/ Mechanical/ Production/
Computer/ Info. Tech./ Electronics / Bio Medical / E & TC) 2014**
Course (CBCS) : SUMMER - 2019

SUBJECT: ENGINEERING MATHEMATICS - I

Day: Thursday
Date: 09/05/2019

S-2019-2525

Time: 10.00 AM TO 01.00 PM
Max Marks. 60

N.B. :

- 1) All questions are **COMPURSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Use of non-programmable calculator is **ALLOWED**.
- 4) Draw neat and labeled diagram **WHEREVER** necessary.

Q.1 a) Find the rank of the following matrix (05)

$$\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$$

b) Examine for linear dependence or independence for the following system of (05)
vectors.

$$x_1 = (3, 1, -4), x_2 = (2, 2, -3), x_3 = (0, -4, 1)$$

OR

Q.1 Find the eigen values and the corresponding eigen vectors of the following (10)
matrix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Q.2 a) A square lies entirely in second quadrant if one of the side join the points -2 (05)
and $2i$ find the complex numbers representing other vertices.

b) If Z_1, Z_2, Z_3 represent vertices of an equilateral triangle, prove that (05)

$$Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$$

OR

Q.2 a) Find the value of $\tanh(\log(\sqrt{5}))$ (05)

b) If $\tan(A + iB) = x + iy$ then prove that $x^2 + y^2 + 2x \cot 2A = 1$ (05)

Q.3 a) If $y = \cos(m \log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$ (05)

b) If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

OR

Q.3 Prove that the value of the n^{th} differential coefficient of $\frac{x^3}{(x^2 - 1)}$ for $x = 0$ is (10)
zero if n is even and $-n!$ if n is odd and greater than 1

P.T.O.

Q.4 a) Evaluate $\lim_{x \rightarrow 0} \frac{e^x \sin x - x - x^2}{x^2 + x \log(1-x)}$ (05)

b) Test the convergence of the series: (05)

$$\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \frac{4}{7.9} + \dots$$

OR

Q.4 a) Test the convergence of the series: (05)

$$1 + \frac{1}{2.3} + \frac{1.3}{2.4.5} + \frac{1.3.5}{2.4.6.7} + \dots$$

b) Find a, b, c if $\lim_{x \rightarrow 0} \frac{ae^x - be^{-x} - cx}{x - \sin x} = 4$ (05)

Q.5 a) If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$ then prove that (05)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

b) If $u = \tan^{-1} \left(\frac{x^3 + y^3}{2x + 3y} \right)$ then prove that (05)

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \sin 2u (2 \cos 2u - 1)$$

OR

Q.5 Find $\frac{dy}{dx} + \frac{d^2 y}{dx^2}$ for $x^4 + y^4 = 5a^2 xy$ (10)

Q.6 a) If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$, $u + v + w = x^2 + y^2 + z^2$ (05)
then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$

b) If the kinetic energy is given by $T = \frac{mv^2}{2}$, find approximately the change in (05)
 T as m changes from 49 to 49.5 and v changes from 1600 to 1590.

OR

Q.6 Examine for minimum and maximum values for (10)
 $\sin x + \sin y + \sin(x + y)$

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