B. Tech. SEM -II (Computer Science & Business Systems) (CBCS 2018 Course) : SUMMER - 2019

SUBJECT : MATHEMATICS – II

Day Wednesday
Date 22/05/2019

S-2019-2519

Time : 10.00 AM To 01.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.
4) Assume suitable data if necessary.

Q.1 a) By using properties of determinants, prove that : **[05]**

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

b) Solve the following system of equations using Cramer's rule : **[05]**

$x-4y-z=11,$

$2x-5y+2z=39,$

$-3x+2y+z=1$

OR

a) If $A=\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C=\begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$, verify that (AB)C = A(BC). **[05]**

b) For $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$, find adjoint (A) and A^{-1}. **[05]**

Q.2 a) Reduce the following matrix A to its normal form and hence find its rank where: **[05]**

$$A=\begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}.$$

b) Apply Gauss - elimination method to solve the equation: **[05]**

$x+4y-z=-5,$

$x+y-6z=-12,$

$3x-y-z=4$

OR

Solve the system of equations by LU decomposition method: **[10]**

$2x_1+2x_2+3x_3=4,$

$4x_1-2x_2+x_3=9,$

$x_1+5x_2+4x_3=3$

Q.3 a) Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on **[05]**

$u=(u_1, u_2)$ and $v=(v_1, v_2)$:

$u+v=(u_1+v_1,\ u_2+v_2)$, $ku=(ku_1,\ 0)$ where k is any real number

i) Compute u + v and ku for u = (2, 4), v = (–3, 5) and K = 7.

ii) Show that V is not a vector space under the given operations.

P.T.O.

b) Determine whether the vectors span R^3. [05]
$V_1 = (2, -1, 3)$, $V_2 = (4, 1, 2)$, $V_3 = (8, -1, 8)$.

OR

a) Find a basis for $\langle S \rangle$, where : [05]

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \\ 3 \end{bmatrix} \right\}$$

b) Let L be the line through the origin in R^2 that is parallel to the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Find the standard matrix of the orthogonal projection onto L and also find the point on L which is closest to the point (7, 1). [05]

Q.4 Let R^3 have the inner product $\langle u, v \rangle = u_1 v_1 + 2u_2 v_2 + 3u_3 v_3$. [10]
Use the Gram-Schmidt process to transform
$u_1 = (1, 1, 1)$, $u_2 = (1, 1, 0)$, $u_3 = (1, 0, 0)$ into an orthonormal basis.

OR

Find the QR – decomposition of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{bmatrix}$. [10]

Q.5 Find Eigen values and Eigen vectors for the matrix. [10]

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

OR

a) Consider the following two bases of R^2: [05]
$S = \{u_1, u_2\} = \{(1,2), (3,5)\}$ and
$S' = \{v_1, v_2\} = \{(1,-1), (1,-2)\}$
Find the change of basis matrix P from S to the "new" basis S'.

b) If $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$, show that A is a Hermitian matrix. [05]

Q.6 Find the singular value decomposition of $A = \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix}$. [10]

OR

What is PCA? Write the objectives of PCA and explain PCA with example. [10]

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