

BACHELOR OF TECHNOLOGY (C.B.C.S.) (2021-COURSE)  
B. Tech. Sem - I Electronics & Communication : WINTER- 2022  
SUBJECT : LINEAR ALGEBRA, CALCULUS & SOLID GEOMETRY

Day : Monday

Time : 10:00 AM-01:00 PM

Date : 9/1/2023

W-24083-2022

Max. Marks : 60

N.B.

- 1) All questions are **COMPULSORY**.
- 2) Figures to the **RIGHT** indicate **FULL** marks.
- 3) Use of non-programmable calculator is **allowed**.
- 4) Assume suitable data **WHEREVER** necessary.

Q.1 Investigate for what values of a & b, the system have (10)

$$2x - y + 3z = 2$$

$$x + y + 2z = 2$$

$$5x - y + az = b$$

- i) No solution
- ii) A unique solution
- iii) An infinite solution.

OR

Q.1 Find the Eigen values and Eigen vectors of the given matrix (10)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}.$$

Q.2 If  $u = \operatorname{cosec}^{-1} \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x^{\frac{1}{3}} + y^{\frac{1}{3}}}}$ , show that  $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{\tan u}{144} (13 + \tan^2 u)$ . (10)

OR

Q.2 If  $f(x, y) = 0$  and  $\phi(y, z) = 0$  then show that  $\frac{\partial f}{\partial y} \frac{\partial \phi}{\partial z} \frac{dz}{dx} = \frac{\partial f}{\partial x} \frac{\partial \phi}{\partial y}$ . (10)

Q.3 Find the maximum and minimum values of  $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ . (10)

OR

Q.3 If  $x = v^2 + w^2$ ,  $y = w^2 + u^2$ ,  $z = u^2 + v^2$ , prove that  $JJ' = 1$ . (10)

Q.4 Find the Fourier series of  $f(x) = x^2$ ,  $x \in (0, 2\pi)$ . (10)

OR

Q.4 Prove that  $\operatorname{erfc}(-x) + \operatorname{erfc}(x) = 2$ . (10)

Q.5 Find the equation of the sphere passing through the points  $(0, 0, 0)$ ,  $(0, -1, 1)$ ,  $(-1, 2, 0)$ ,  $(1, 2, 3)$ . (10)

OR

Q.5 Show that the plane  $2x - 2y + z + 12 = 0$  touch the sphere  $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$  and find the point of contact. (10)

Q.6 Show that  $\int_0^1 \int_{x^2}^{2-x} y dy dx = \frac{16}{15}$ . (10)

OR

Q.6 Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$ . (10)