

BACHELOR OF TECHNOLOGY (CBCS - 2023)
 B. Tech. Sem-II Computer Science & Business Systems : WINTER : 2024
 SUBJECT: LINEAR ALGEBRA

Day : Saturday
 Date : 30/11/2024

W-27707-2024

Time : 10:00 AM-01:00 PM
 Max. Marks : 60

N.B.

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Assume suitable data **WHEREVER** necessary.
- 4) Draw neat diagrams **WHEREVER** necessary.

Q.1 Solve the following equation by using Cramer's rule (10)
 $x + y + z = 6, 3x + 3y + z = 12, 2x + 3y + 2z = 14.$

OR

Q.1 If $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0.$ (10)

Then without expanding the determinant prove that $xyz = -1$. Where x, y, z are unequal.

Q.2 Find the rank of $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$. (10)

OR

Q.2 Solve $4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21.$ (10)

Q.3 Determine whether or not the following set S of vectors form a basis of R^4 . $S = \{(1,1,1,1), (1,2,3,2), (2,5,6,4), (2,6,8,5)\}.$ (10)

OR

Q.3 Let W be the subspace of R^4 spanned by vectors $u_1 = (1, -2, 5, -3), u_2 = (2, 3, 1, -4), u_3 = (3, 8, -3, -5).$ Find a basis and dimension of W . (10)

Q.4 Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace U of R^4 spanned by $v_1 = (1, 1, 1, 1), v_2 = (1, 2, 4, 5), v_3 = (1, -3, -4, -2).$ (10)

OR

Q.4 Find a QR factorization of $A = \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$. (10)

Q.5 Find the Eigen values and the corresponding Eigen vectors of (10)

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

OR

Q.5 Determine whether the given matrix is Hermitian $A = \begin{bmatrix} 4 & -3 & 5 \\ -3 & 2 & 1 \\ 5 & 1 & -6 \end{bmatrix}$. (10)

Q.6 Find a singular value decomposition of $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. (10)

OR

Q.6 Give the following data, use principal component analysis to reduce the dimension from 2 to 1. (10)

Feature	Example1	Example2	Example3	Example4
X	4	8	13	7
Y	11	4	5	14
