B - Tech - sem - I , (2014, Course) , IT - winter - 2018

B.Tech. SEM -I (Chemical/ Civil/ Electrical/ Mechanical/ Production/ Computer/ Info. Tech./ Electronics / Bio Medical / E & TC) 2014 Course (CBCS) : WINTER - 2018

SUBJECT : ENGINEERING MATHEMATICS – I

Day : Thursday
Date : 22/11/2018

Time : 10.00 AM TO 01.00 PM
Max. Marks : 60

W-2018-2260

N. B. :

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.
4) Assume suitable data, if necessary.

Q. 1 **a)** Reduce the following matrix to its normal form hence find its rank: **(05)**

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

b) Examine for consistency and if consistent then solve it. **(05)**

$$x + y + z = 6$$
$$2x + y + 3z = 13$$
$$5x + 2y + z = 12$$
$$2x - 3y - 2z = -10$$

OR

Find eigen values and eigen vectors for the following matrix: **(10)**

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Q. 2 **a)** If P is real and complex number $\frac{1+i}{2+ip} + \frac{2+3i}{3+i}$ is represented on Aryand's diagram by a point on a line $y = x$, show that $P = -5 \pm \sqrt{21}$ **(05)**

b) Two opposite vertices of a square are represented by complex numbers $(9 + 2i)$ and $(-5 + 10i)$. Find the complex number representing the other two vertices of the square. **(05)**

OR

a) If $\tan hx = \frac{2}{3}$, find the value of $\cos h\, 2x$. **(05)**

b) If $\tan\left(\frac{x}{2}\right) = \tanh\left(\frac{y}{2}\right)$. Prove that $\sinh x = \tan x$ **(05)**

P. T. O.

Q. 3 **a)** Find n[th] derivatives of : $y = \dfrac{x^3}{(x-1)(x-2)}$ **(05)**

b) Prove that: **(05)**

$$e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + ---$$

OR

If $y = \left(\sin^{-1} x\right)^2$, Prove that $\left(1-x^2\right) y_{n+2} - (2n+1)\, x y_{n+1} - n^2 y_n = 0.$ **(10)**

Q. 4 **a)** Evaluate: $\lim\limits_{x \to 1} \dfrac{\sqrt{1+x} - \sqrt{1-x^2}}{\sqrt{1-x} - \sqrt{1-x^2}}$ **(05)**

b) Find a and b if $\lim\limits_{x \to 0} \dfrac{a \sinh x + b \sin x}{x^3} = \dfrac{5}{3}$ **(05)**

OR

a) Test for convergence the series whose n[th] term is $\sqrt{\dfrac{2^{n-1}}{3^{n-1}}}$ **(05)**

b) Test for convergence the series $\sum\limits_{n=0}^{\infty} \dfrac{1}{n^2+1}$ **(05)**

Q. 5 **a)** If $u = \tan\left(y + ax\right) - \left(y - ax\right)^{3/2}$ then verify $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$. **(05)**

b) If $x^3 + y^3 = 3ax^2$ then find $\dfrac{\partial^2 y}{\partial x^2}$ **(05)**

OR

Find: **(10)**

$$x^2 W_{xx} + 2xy\, W_{xy} + y^2 W_{yy} \text{ if } W = \operatorname{cosec}^{-1} \sqrt{\frac{\sqrt{x} + \sqrt{y}}{\sqrt[3]{x} + \sqrt[3]{y}}} + \tan^{-1}\left(\frac{x^3 + y^3}{2x + 3y}\right)$$

Q. 6 Discuss the maxima and minima of the following function: **(10)**

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$

OR

a) If $u = xyz,\ v = x^2 + y^2 + z^2,\ w = x + y + z$ **(05)**

Find: $\dfrac{\partial x}{\partial u}$

b) Examine for functional dependence of $u = \dfrac{x-y}{x+y},\ v = \dfrac{x+y}{x},$ **(05)**

if functionally dependent find the relation between them.

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