# BACHELOR OF TECHNOLOGY (CBCS - 2023)

## B. Tech. Sem-I Computer Science & Business System : WINTER : 2023

## SUBJECT : INTRODUCTORY TOPICS IN STATISTICS, PROBABILITY & CALCULUS

Day : Monday

Date : 04-12-2023

**W-27622-2023**

Time : 10:00 AM-01:00 PM

Max. Marks : 60

**N.B :**

1) All questions are **Compulsory**.
2) Figures to the right indicates **Full** marks.
3) Use to the non-programmable **CALCULATOR** is allowed.
4) Assume suitable data **WHEREVER** necessary.

**Q1** Find eigen values and eigen vectors for the following matrices:

$$\begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$$ **(10)**

**OR**

**Q1.** Test for consistency and solve:

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5.$ **(10)**

**Q2.** Find all the roots of $x^8 - 1$ and identify the roots which are also the roots of $x^3+x^2+x+1$ and hence show that $(1-\omega)(1-\omega^2)(1-\omega^3) = 4$ which $\omega = e^{i\pi/2}$. **(10)**

**OR**

**Q2.** If $\sin\alpha + \sin\beta + \sin\gamma =$ If $\cos\alpha + \cos\beta + \cos\gamma = 0$, prove that

(i) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$. **(10)**

**Q3.** Find the $n^{th}$ differential coefficients of the following functions: **(10)**

Sin 2x cos 3x

**OR**

**Q3.** Expand $x^4-3x^3+2x^2-x+1$ in power of (x-3). **(10)**

04123-m-coe-mumbai

**Q4.** Prove that $\lim_{x\to\infty}\left(\frac{1^{1/x}+2^{1/x}+3^{1/x}}{3}\right)^{3x}=6.$ **(10)**

**OR**

**Q4.** Test the convergence or divergence of the following series: $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{9}}, \frac{1}{\sqrt{28}}, \frac{1}{\sqrt{65}}, \cdots$ **(10)**

**Q5.**

If $z = f(x, y)$ $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\left(\frac{\partial y}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$ **(10)**

**OR**

**Q5.** Find the possible percentage error in computing the parallel resistance r of the three resistances $r_1, r_2, r_3$ from the formula $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$ if $r_1, r_2, r_3$ are each in error by plus 1.2%. **(10)**

**Q6.** If $x=v^2+w^2, y=w^2+u^2, z=u^2+v^2$, prove that $JJ'=1$. **(10)**

**OR**

**Q6.** If $x = u\cos v$, $y = u\sin v$, prove that $\frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(u,v)}=1$ **(10)**

* * * * *

04123-m-coe-mumbai