BACHELOR OF TECHNOLOGY (C.B.C.S.) (2020 COURSE)
B.Tech.Sem - IV CIVIL : WINTER- 2022
SUBJECT : VECTOR CALCULUS & DIFFERENTIAL EQUATIONS

Day : Thursday | Time : 02:30 PM-05:30 PM

Date : 24-11-2022 | **W-24371-2022** | Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Draw neat and labeled diagram wherever necessary.
4) Assume suitable data if necessary.

Q.1 Solve by method of variation of parameters **[10]**

$$\frac{d^2y}{dx^2} - y = e^{-x}\sin\left(e^{-x}\right) + \cos\left(e^{-x}\right)$$

OR

Q.1 Solve : $(2x+1)^2\frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$. **[10]**

Q.2 A body weighing W = 20N is hung from a spring. A pull of 40 N will stretch the spring to 10 cm. The body is pulled down to 20 cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position in time t seconds, the maximum velocity and period of oscillation. **[10]**

OR

Q.2 A cantilever beam of length l and weighing W *l*b/unit is subjected to a horizontal compressive force P applied at the free end. Taking the origin of the free end and y axis upwards, establish the differential equation of the beam and hence find the maximum deflection. **[10]**

Q.3 If $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ represents the vibrations of a string of length l fixed at both ends, find the solution with boundary conditions **[10]**

a) $y(0, t) = 0$, b) $y(l, t) = 0$

and initial conditions

c) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$ d) $y(x,0) = k\left(lx - x^2\right),\ 0 \le x \le l$

OR

Q.3 Solve : $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ if **[10]**

a) $u(o, t) = 0$ b) $u_x(l, t) = 0$

c) $u(x, t)$ is bounded d) $u(x,0) = \frac{u_0 x}{l}$ for $0 \le x \le l$

Q.4 Show that $\overline{F} = \left(ye^{xy}\cos z\right)\hat{i} + \left(xe^{xy}\cos z\right)\hat{j} - \left(e^{xy}\sin z\right)\hat{k}$ is irrotational. **[10]**

Find the corresponding scalar ϕ such that $\overline{F} = \nabla\phi$.

P.T.O.

OR

Q.4 Show that: **[10]**

a) $\nabla^4\left(r^2 \log r\right) = \frac{6}{r^2}$

b) $\nabla.\left[r\nabla\left(\frac{1}{r^n}\right)\right] = \frac{n(n-2)}{r^{n+1}}$

Q.5 Find work done by the force **[10]**
$\left(x^2 - y - z\right)\hat{i} + \left(y^2 - zx\right)\hat{j} + \left(z^2 - xy\right)\hat{k}$ in taking a particle from (1, 1, 1) to (3, –5, 7).

OR

Q.5 Apply stoke's theorem to calculate $\int_c 4ydx + 2zdy + 6ydz$, where C curve of **[10]**
intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$.

Q.6 Two examiners A and B independently award marks to given students: **[10]**

R. No. :	1	2	3	4	5	6	7
Marks by A	40	44	28	30	44	38	31
Marks by B	32	39	26	30	38	34	28

Obtain the equation of regression lines. If examiner A awards 36 marks to roll No. 8, what would be the marks expected to be awarded by examiners B to the same candidate?

OR

Q.6 On an average a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have three or less defectives? **[10]**

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