BACHELOR OF TECHNOLOGY (CBCS - 2023)
B. Tech. Sem-III Computer Science & Engineering AI & ML : WINTER : 2024
SUBJECT: DISCRETE MATHEMATICS

Day : Tuesday
Date : 03/12/2024

W-29207-2024

Time : 10:00 AM-01:00 PM
Max. Marks : 60

N.B.

1) All questions are **COMPULSORY**.
2) Figures to **RIGHT** indicate **FULL** marks.

Q.1 What are predicates & Quantifiers? Explain different types of quantifiers **(10)**

Let p and q be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence.

a) $\neg p$ b) $\neg p \wedge q$ c) $q \rightarrow p$ d) $p \leftrightarrow q$

OR

Use mathematical induction to show that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integers n.

What are Conditional and Biconditional Statements. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Q.2 Explain the terms a) Cardinality of Set b) Power Set **(10)**

Draw the Venn diagrams for each of these combinations of the sets A, B, and C.
a) $A \cap B \cap C$ b) $(A - B) \cup (A - C) \cup (B - C)$

Evaluate these quantities. a) 144 mod 7 b) 155 mod 19

OR

State fundamental theorem of number theory.

Determine whether the integers in each of these sets are pairwise relatively prime: 21, 34, 55.

Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find a) $A \times B \times C$. b) $C \times B \times A$. c) $B \times B \times B$.

Q.3 Explain the different types of closure of Relation. (10)

Which of these relations on {0, 1, 2, 3} are equivalence relations?

a) {(0, 0), (1, 1), (2, 2), (3, 3)} b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

OR

Explain the concept: a) One – One Function b) Onto Function

Find the transitive closures of these relations on {1, 2, 3, 4}.

{(1, 2), (2,1), (2,3), (3,4), (4,1)}

Q.4 Explain the concept in counting: a) Sum Rule b) Product rule (10)

How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points

OR

State & explain Pigeon Hole Principle with example.

How many different bit strings of length seven are there?

Q.5 Consider the tree with root v_0 shown below. Answer the following questions: (10)

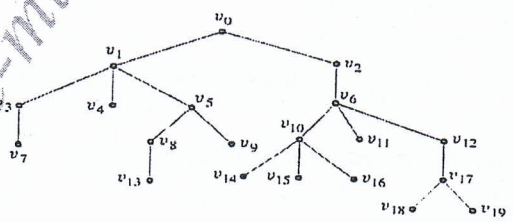
a. What is the level of v_5?

b. What is the height of this rooted tree?

c. What are the children of v_{17}?

d. What is the parent of v_2?

e. What is the degree of vertex v_6



Explain the terms : a) Trail b) Path c) Circuit d) Simple Circuit.

OR

What is Tree? Explain rooted tree in details.

Determine which of the following graphs have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one

Q.6 Explain the terms a) Monoid b) Semi Group **(10)**

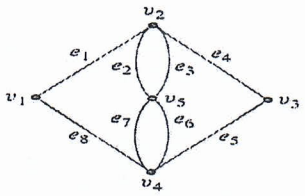
Prove that $G = \{-1, 1, -i, i\}$ is a group under usual multiplication

OR

Explain the concept a) Ring b) Field c) Integral domain

Let $G = \{0, 1, 2, 3, 4, 5\}$ find left and right cosets of $H = \{0, 3\}$. Is H a normal subgroup of G.

12.



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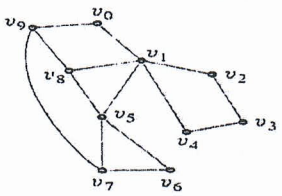

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