

B. Tech. SEM -I (Computer Science & Business Systems) (CBCS 2018 Course) : SUMMER - 2019

SUBJECT : MATHEMATICS – I

Day : Thursday
Date : 09/05/2019

S-2019-2513

Time : 10.00 AM To 01.00 PM
Max. Marks : 60

N. B. :

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Use of non-programmable calculator is **ALLOWED**.

Q. 1 Evaluate the following integrals:

a) $\iint_R y \, dx \, dy$, where R is $y = x^2$, $x + y = 2$, $x = 0$ in first quadrant. (05)

b) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{dx \, dy}{\sqrt{1-x^2-y^2}}$. (05)

OR

Change the order of integration in the double integral (10)

$$\int_0^a \int_{-a+\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} f(x, y) \, dx \, dy.$$

Q. 2 a) Find the volume of the region enclosed by the cone (05)

$z = \sqrt{x^2 + y^2}$ and paraboloid $z = x^2 + y^2$.

b) Find the total area included between the two cardioids $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$. (05)

OR

Find the area inside the cardioid $r = 2a(1 + \cos\theta)$ and outside the parabola (10)

$$r = \frac{2a}{1 + \cos\theta}.$$

Q. 3 a) Among the first 1000 positive integers determine the integers which are not (05)
divisible by 5 nor by 7 nor by 9.

b) Construct the truth table for the following statement: (05)
 $(\sim q \rightarrow \sim p) \rightarrow (p \rightarrow q)$.

OR

a) Determine whether the relation (05)
 $R = \{(a, b) \in R, a - b \leq 1, \text{ on the set of positive integers}\}$ is an equivalence
relation.

b) Construct the truth table for the following statement: (05)
 $p \leftrightarrow (\sim p \vee \sim q)$.

P. T. O.

Q. 4 a) Reduce the expression (05)

$$\overline{XY} + \overline{X} + XY.$$

b) Design a circuit to realize the following: (05)

$$F(A, B, C) = AB + A\overline{C} + \overline{B}AC.$$

OR

What is the simplified Boolean equation for the function: (10)

$$F(A, B, C, D) = \sum(7, 9, 10, 11, 12, 13, 14, 15).$$

Q. 5 a) Consider the binary operation $*$ on I_+ (the set of positive integers) defined by (05)

$$a * b = \frac{ab}{2} \quad \forall a, b \in I_+. \text{ Determine the identity for the binary operation } *, \text{ if exists.}$$

b) Let $(G, *)$ be a group, where G is a set having elements $\{0, 1\}$ and $*$ is a (05)
binary operation. Also, let $H = \{1\}$ is a subgroup of G . Determine all the left
coset of H in G .

OR

Consider an algebraic system $(Q, *)$, where Q is the set of rational numbers (10)

and $*$ is a binary operation defined by $a * b = a + b - ab \quad \forall a, b \in Q$.

Determine whether $(Q, *)$ is a group.

Q. 6 a) Determine the discrete numeric functions corresponding to the following (05)
generation function:

$$A(Z) = \frac{2 + 3Z - 6Z^2}{1 - 2Z}$$

b) Prove the following by mathematical induction: (05)

$$1.2 + 2.3 + 3.4 + \dots + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3} \right].$$

OR

a) Solve: $a_r - a_{r-1} - 6a_{r-2} = -30$ given $a_0 = 20, a_1 = -5$. (05)

b) Show that for any integer n , (05)

$$11^{n+2} + 12^{2n+1} \text{ is divisible by } 133.$$

* * * * *