

Day : Thursday  
Date : 09/05/2019

S-2019-2561

Time : 02.30 PM TO 05.30 PM  
Max. Marks : 60

N. B. :

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Draw neat and labeled diagram **WHEREVER** necessary.
- 4) Use of non-programmable calculator is **ALLOWED**.
- 5) Assume suitable data, if necessary.

Q.1 Solve:  $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z - a\sqrt{x^2 + y^2 + z^2}}$  (10)

OR

Solve: (10)

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0$$

given that  $x = 0; y = 0$  at  $t = 0$ .

Q.2 Evaluate: (10)

$$\int_c \frac{(4z^3 - 1) dz}{(z^2 - 1)(z + 1)^3(z - 4)} \quad \text{where } c: |z| = \frac{5}{2}.$$

OR

Find the bilinear transformation which maps the points  $(-i, 0, 2 + i)$  of the Z-plane onto the points  $(0, -2i, 4)$  of the W-plane. (10)

Q.3 a) Find the z-transform of  $(k + 1)(k + 3)4^k; k \geq 0$ . (05)

b) Find Fourier cosine transform of  $x^{n-1}$  (05)

OR

Find Fourier transform of  $\frac{e^{-ax}}{x}$  and hence evaluate  $\int_0^{\infty} \tan^{-1}\left(\frac{x}{a}\right) \sin x dx$  (10)

P. T. O.

Q. 4 a) Evaluate, using Laplace transform: (05)

$$\int_0^{\infty} e^{-at} \frac{\sin t}{t} dt.$$

b) Find : (05)

$$L^{-1} \left[ \log \left( \frac{s+4}{s+7} \right) \right]$$

OR

Solve using inverse Laplace transform : (10)

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \text{ if } x(0) = 1, x\left(\frac{\pi}{2}\right) = -1.$$

Q. 5 a) Find the function  $g(r)$  so that  $g(r)\vec{r}$  is solenoidal. (05)

b) Find the value of  $\nabla^2(r^n \log r)$ . (05)

OR

Show that vector field : (10)

$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$  is irrotational. Find the scalar potential  $\psi$  such that  $\vec{F} = \nabla\psi$

Q. 6 Evaluate :  $\iint_S (x^3\vec{i} + y^3\vec{j} + z^3\vec{k}) \cdot d\vec{s}$ , where  $S$  is the surface of the sphere  $x^2 + y^2 + z^2 = 9$ . (10)

OR

State and prove Green's theorem. (10)

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