

BACHELOR OF TECHNOLOGY (C.B.C.S.) (2021-COURSE)

B. Tech. Sem - I Robotics & Automation Engineering : WINTER- 2022
SUBJECT : LINEAR ALGEBRA, CALCULUS & COMPLEX VARIABLES

Day : Monday

Time : 10:00 AM-01:00 PM

Date : 9/1/2023

W-24117-2022

Max. Marks : 60

N.B.

- 1) All questions are **COMPULSORY**.
- 2) Figures to the **RIGHT** indicate **FULL** marks.
- 3) Use of non-programmable calculator is **allowed**.
- 4) Assume suitable data **WHEREVER** necessary.

Q.1 Reduce the matrix to normal form and hence find its rank (10)

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 4 & 0 & -1 \\ -1 & 0 & -2 & 7 \end{bmatrix}$$

OR

Q.1 Find Eigen values and Eigen vectors for the following matrix (10)

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

Q.2 Find $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$ in the following case $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$ (10)

OR

Q.2 If $(\cos x)^y = (\sin y)^x$ then find $\frac{dy}{dx}$. (10)

Q.3 Verify whether following field is irrotational find corresponding scalar ϕ such that $\bar{F} = \nabla \phi$ where $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$. (10)

OR

Q.3 Find curl curl \bar{F} at $(0,1,2)$ where $\bar{F} = x^2 y \bar{i} + xyz \bar{j} + z^2 y \bar{k}$ (10)

Q.4 Verify Green's theorem for $\bar{F} = x\bar{i} + y^2 \bar{j}$ over the first quadrant of the circle $x^2 + y^2 = a^2$. (10)

OR

Q.4 Find work done by the force $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ in taking particle from $(1,1,1)$ to $(3,-5,7)$. (10)

Q.5 If $v = 3x^2 y - y^3$ find its harmonic conjugate u . find $f(z) = u + iv$ in terms of z . (10)

OR

Q.5 Find the condition under which $u = ax^3 + bx^2 y + cxy^2 + dy^3$ is harmonic. (10)

Q.6 Evaluate $\int \frac{4z^2 + z}{(z-1)^2}$ where c is $|z-2| = 2$. (10)

OR

Q.6 Find the bilinear transformation which sends the points $1, i, -1$ from z -plane into. (10)
