

**BACHELOR OF TECHNOLOGY (CBCS - 2023)**  
**B. Tech. Sem-III Computer Science & Engineering AI & ML : WINTER: 2025**  
**SUBJECT: DISCRETE MATHEMATICS**

Day : Tuesday  
Date : 09/12/2025

W-29207-2025

Time : 10:00 AM-01:00 PM  
Max. Marks : 60

**N.B.**

- 1) All questions are **COMPULSORY**.
- 2) Figures to the **RIGHT** indicate **FULL** marks.
- 3) Assume suitable data **WHEREVER** necessary.
- 4) Draw neat diagrams **WHEREVER** necessary.

**Q.1** Define an universal quantifiers and an existential quantifiers. (10)  
Let  $A = \{1, 2, 3, 4, 5\}$ . Write the following statement in proper English and determine the truth value

- i)  $(\exists x \in A) (x + 3 = 10)$
- ii)  $(\forall x \in A) (x + 3 < 10)$
- iii)  $(\exists x \in A) (x + 3 < 5)$
- iv)  $(\forall x \in A) (x + 3 \leq 7)$

**OR**

**Q.1** Prove by mathematical Induction that,  $n^4 - 4n^2$  is divisible by 3 for all  $n \geq 2$ . (10)

**Q.2** Explain the following terms with example (10)  
a) Universal Set  
b) Power Set  
c) Subset  
d) Cardinality of a set.

Draw a venn diagram for each of these combinations of the set A, B & C  
a)  $A \cap B \cap C$                       b)  $(A - B) \cup (A - C) \cup (B - C)$

**OR**

**Q.2** i) State the fundamental theorem of number theory. (10)  
Determine whether the integers in each of these sets are pairwise relatively prime:  
21, 34, 55.

ii) Let  $A = \{a, b, c\}$ ,  $B = \{x, y\}$  &  $C = \{0, 1\}$   
Find a)  $A \times B \times C$   
b)  $C \times B \times A$   
c)  $B \times B \times B$ .

**Q.3** What is an equivalence relation? Explain with an example (10)  
Let  $X = \{1, 2, \dots, 7\}$  &  $R = \{(x, y) / x - y \text{ is divisible by } 3\}$   
Show that R is an equivalence relation. Draw the graph of R.

**OR**

**Q.3** Explain the concept of a) Injective function, b) Surjective function c) bijective function with an example. (10)

Let  $f(x) = 2x + 3$ ,  $g(x) = 3x + 4$ ,  $h(x) = 4x$   
for  $x \in R$ , where  $R =$  Set of real nos.  
Find  $gof, fog, foh, hof$  &  $gofoh$ .

**Q.4** Explain the concept in counting a) Sum rule principle b) Product rule principle. (10)  
Find how many integers between 1 & 60 are not divisible by 2 nor by 3 and nor by 5?

**P.T.O.**

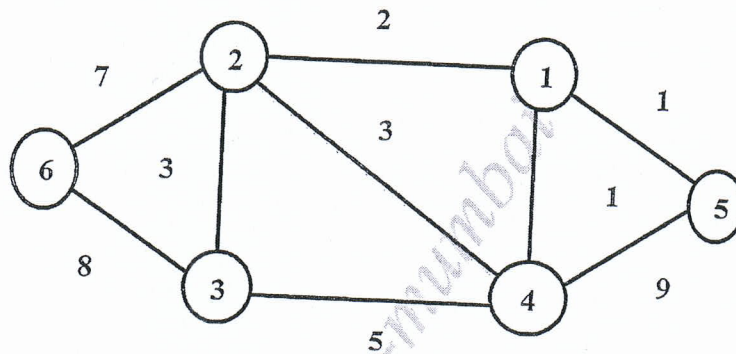
OR

Q.4 State and explain the Pigeon Hole principle with example. (10)  
Show that, if any 11 numbers are chosen from the set  $\{1, 2, \dots, 20\}$  then one of them will be a multiple of another.

Q.5 What is Tree? Explain a binary tree with an example. (10)  
Prove that in any binary tree  $T$  of  $n$  vertices the number of pendent vertices is equal to  $(n+1)/2$ .

OR

Q.5 Construct minimum cost spanning tree for following graph using Prim's method and Kruskal's method. (10)



Q.6 Explain the terms: a) Monoid, b) Semi-group. (10)  
Prove that  $G = \{1, -1, i, -i\}$  is a group under usual multiplication.

OR

Q.6 Explain the concept of a) Ring b) Field c) Integral domain (10)  
Let  $G = \{0, 1, 2, 3, 4, 5\}$ . Find the left and right cosets of  $H = \{0, 2\}$ .  
Is  $H$  a normal subgroup of  $G$ .

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