

**BACHELOR OF TECHNOLOGY (CBCS) (2021-COURSE)**  
**B. Tech. Sem - III Computer Science & Engineering : WINTER : 2024**  
**SUBJECT: DISCRETE MATHEMATICAL STRUCTURES**

Day : Thursday  
Date : 05/12/2024

W-25311-2024

Time : 10:00 AM-01:00 PM  
Max. Marks : 60

**N.B.**

- 1) All questions are **COMPULSORY**.
- 2) Figures to the **RIGHT** indicate **FULL** marks.
- 3) Assume suitable data **WHEREVER** necessary.
- 4) Draw neat labeled diagrams **WHEREVER** necessary.

- Q.1** What are some fundamental methods of proofs used in mathematics to establish the truth of mathematical statement and theorems? Explain the following in detail with example. (10)
- i. Direct Proof.
  - ii. Proof by Contradiction.
  - iii. Proof by Contrapositive.
- OR**
- Q.1** What is a proof by Mathematical Induction? Show for any natural number  $n$  by Mathematical Induction: (10)  
 $1! + (2 \times 2!) + (3 \times 3!) + \dots + (n \times n!) = (n + 1)! - 1$ .
- Q.2** Explain the Principle of Inclusion and Exclusion. Solve the following: (10)  
Among 50 patients admitted to a hospital, 25 are diagnosed with pneumonia, 30 with bronchitis and 10 with both pneumonia and bronchitis. Determine:
- i. The number of patients diagnosed with pneumonia or bronchitis.
  - ii. The number of patients not diagnosed with pneumonia or bronchitis.
- OR**
- Q.2** Explain what the prime numbers are and their special properties with suitable examples. Also explain the significance of prime numbers in Discrete Mathematics. (10)
- Q.3** Let  $f: R \rightarrow R$  be a function defined as  $f(x) = x^2$  and  $g: R \rightarrow R$  be a function defined as  $g(x) = \sqrt{x}$ . (10)
- i. Determine whether each function  $f$  and  $g$  is injective, surjective or bijective.
  - ii. Find the composition  $f \circ g$  and  $g \circ f$ .
- OR**
- Q.3** Define a relation and explain its properties. Explain different types of relations with suitable examples. (10)
- Q.4** Explain how the concept of sets, functions and relations can be applied to model the process of error detection and recovery using parity checks. (10)
- OR**
- Q.4** Define a function and related terminologies such as image, domain, co-domain with suitable examples. Explain different types of functions. Solve the following: (10)
- i. Show that the function  $f: R \rightarrow R$ , given by  $f(x) = 2x$  is one-one and onto.
  - ii. If  $f: R \rightarrow R$ ;  $f(x) = 2x + 7$  is a bijective function then find Inverse of  $f$ .
- Q.5** What is a Recurrence Relation? Explain first Order and Second Order Recurrence Relations. Consider the sequence  $a_0, a_1, a_2, \dots$  defined by the recurrence relation  $a_{n+2} = 3a_{n+1} - 2a_n$  for  $n \geq 0$ , with initial condition  $a_0 = 1$  and  $a_1 = 2$ . Calculate the first five terms of the sequence. (10)
- OR**
- Q.5** Explain with example and suitable formulae what are Permutations and Combinations. Solve the following: (10)
- i. How many ways are there to distribute four distinct balls evenly between two distinct boxes (two balls to go in each box)?
  - ii. How many permutations are there of 5 things taken 3 at a time?
- Q.6** What is a graph? Explain different representation and operations on graphs. (10)
- OR**
- Q.6** What is a Binary Tree? Explain its properties and any two applications of binary trees. (10)

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