

BACHELOR OF TECHNOLOGY (CBCS - 2023)  
 B. Tech. Sem-II Computer Science & Business Systems : WINTER : 2024  
 SUBJECT: LINEAR ALGEBRA

Day : Saturday  
 Date : 30/11/2024

W-27707-2024

Time : 10:00 AM-01:00 PM  
 Max. Marks : 60

**N.B.**

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Assume suitable data **WHEREVER** necessary.
- 4) Draw neat diagrams **WHEREVER** necessary.

**Q.1** Solve the following equation by using Cramer's rule (10)  
 $x + y + z = 6, 3x + 3y + z = 12, 2x + 3y + 2z = 14.$

OR

**Q.1** If  $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0.$  (10)

Then without expanding the determinant prove that  $xyz = -1$ . Where  $x, y, z$  are unequal.

**Q.2** Find the rank of  $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}.$  (10)

OR

**Q.2** Solve  $4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21.$  (10)

**Q.3** Determine whether or not the following set  $S$  of vectors form a basis of  $R^4$ .  $S = \{(1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4), (2, 6, 8, 5)\}.$  (10)

OR

**Q.3** Let  $W$  be the subspace of  $R^4$  spanned by vectors  $u_1 = (1, -2, 5, -3), u_2 = (2, 3, 1, -4), u_3 = (3, 8, -3, -5).$  Find a basis and dimension of  $W$ . (10)

**Q.4** Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace  $U$  of  $R^4$  spanned by  $v_1 = (1, 1, 1, 1), v_2 = (1, 2, 4, 5), v_3 = (1, -3, -4, -2).$  (10)

OR

**Q.4** Find a QR factorization of  $A = \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}.$  (10)

**Q.5** Find the Eigen values and the corresponding Eigen vectors of (10)

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}.$$

OR

**Q.5** Determine whether the given matrix is Hermitian  $A = \begin{bmatrix} 4 & -3 & 5 \\ -3 & 2 & 1 \\ 5 & 1 & -6 \end{bmatrix}.$  (10)

**Q.6** Find a singular value decomposition of  $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$  (10)

OR

**Q.6** Give the following data, use principal component analysis to reduce the dimension from 2 to 1. (10)

Feature	Example1	Example2	Example3	Example4
X	4	8	13	7
Y	11	4	5	14

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